2 차원에서 미디어 이송 시스템의 동적 거동 해석을 위한 수치적인 모델 개발

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Development of Numerical Models for Dynamics Analysis of Media Transport System in 2D

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ABSTRACT

Recently such media transport systems as printers, copiers, fax, ATMs, and so on have been widely used. Especially paper feeding mechanism is an important key technology in printers, copying machines, photo developing machines, automated teller machines and so on. A number of experiments were essential to determine kinematic mechanism, parts dimensions and materials on designing and improving these machines. It requires a lot of time and cost. To shorten the time, reduce the cost and strengthen a machine performance, it is strongly required to develop a numerical simulator which analyses paper feed and separation process. In this paper, numerical models of a flexible sheet such as paper or film, a roller and a guide will be introduced. One of the most important numerical models is a flexible sheet. The flexible sheet undergoes large deformations with linear material properties. The most popular 2D approximation of the proper behavior of a sheet is that the sheet has been a series of rigid bars connected by revolute joints and rotational spring dampers. The advantage of this approach is good visual appearance of the sheet under severe bending conditions. Paper feed together with paper separation is also important technology in handling flexible media. Paper is fed by a contact and friction mechanism on rollers or guides. For there are so many contact pair bodies, a contact searching time is so long for each time. So, efficiently to detect a contact phase, a global detecting method that a bounding box is divided into several pieces in global coordinate system is presented in this paper. The method has an advantage that the number of contact searching can be smaller than other methods for a system in which the position of most of rollers and guides are fixed on a point of a base body such as the ground or one carrier body. The proposed numerical models will make it possible to check for the potential for jamming given different sheet size, weight and stiffness, different sheet properties due to temperature and humidity extremes, different sheet velocity due to misalignment of drive-driven roller sets and differences of roller velocities due to gap, wear or etc.

Key Words : Media Transport System, Paper Feeding Machines, Flexible Sheet, Global Detecting Method

1. INTRODUCTION

Recently the media transport systems, such as printers, copiers, fax, ATMs, cameras, film develop machines, etc., have been widely used and being developed rapidly. Especially, in the development of those systems, the media feeding mechanism for paper, film, money, cloth etc., is an important key technology for the design and development of the media transport systems. Tedious and iterative experimental trial and errors methods have been essential way to determine kinematic mechanisms of parts dimensions, and materials, etc for the media machine developers. Since the iterative trial & error methods are truly inefficient, in order to shorten the time, reduce the cost, and improve the machine performance, it has been absolutely required to develop the computer simulation tool, which analyses the paper feeding and separation process.

Cho and Choi [1] developed a computational modeling techniques for two dimensional film feeding mechanisms. The flexible film is divided by several thin rigid bodies which are connected by revolute joints and rotational spring dampers. The primitive computer implementation methods for contact search algorithms are presented. Diehl [2], [5] presented the local static mechanics of electrometric nip system for media transport system. The nonlinear finite element method and experimental measurement techniques are used to investigate the large deformable rollers. Several unique phenomena, such as skewing sheet, etc., of nip feeding system are well described in this research. Ashida [3] suggested the computer modeling techniques for the design and analysis of film feeding mechanisms. The primitive dynamic analysis of two dimensional film feeding models are presented by using commercial computer program. The paper feed mechanism with friction pad system is investigated by Yanabe [4] by using commercial nonlinear FEA program. It show the local separation phenomenon between papers and roller, and proved very good agreement with experimental measurements. Shin [6], [7] developed web simulation and design tools using roll tensions. They show that the control of tensions of each segment is the key design factors for web system.

In this investigation, a numerical modeling method and dynamic analysis of the two dimensional flexible sheet for thin flexible media materials such as paper, film, etc., and their roller and guide contacts are suggested by using multibody dynamic techniques. Since the flexible sheet undergoes large deformation with assumed linear material properties, the flexible sheet has been modeled as a series of thin rigid bars connected by revolute joints with rotational spring dampers force elements. It shows good visual appearance of the sheet under severe bending conditions. An efficient contact search and force analysis between sheet and rollers, and guides are developed and implemented numerically. The sheet is fed by contact and friction forces when it contacts with rotating rollers or guides. In order to detect a contact phase efficiently, the bounding box method is used in this contact search algorithm. The method has an advantage that the number of contact search can be smaller than conventional methods for a system in which the position of rollers and guides are fixed on a point of a base body. The proposed numerical models for media transport systems will make it possible to confirm the potential problems of jamming by given different sheet size, weight, stiffness, temperature, humidity extremes, sheet velocity due to misalignment of drive-driven roller sets, and roller velocities due to gap, wear or etc.

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2. TWO DIMENSIONAL FLEXIBLE MULTIBODY SHEET

In general, there are two methods to build a thin 2-D flexible sheet for dynamic analysis. One is to employ beam element at discretized sheet body, and the other is small rigid bar interconnected by revolute joint with rotational spring-damper forces. In this investigation, the second method is used and proposed the modeling techniques.

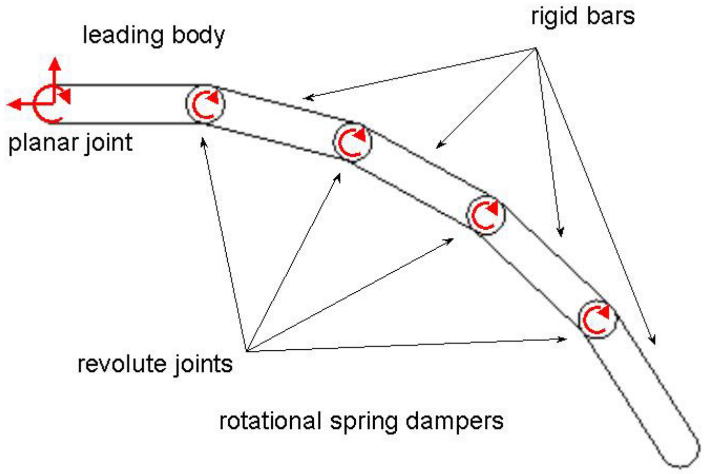

Figure 1 Modeling definition of a two dimensional flexible sheet

Several research works show that the most efficient way to model two-dimensional approximation of the proper behavior of a sheet can be a series of rigid bars connected by revolute joints and rotational spring-dampers as shown in Figure 1 [1], [3]. The sheet is divided into a number of rigid bars with mass. The mass and inertia moment of each rigid bar can be defined as follows

$$m = \rho\, L_s\, t_s \tag{1}$$

$$I_{zz} = m\frac{(t_s^2 + L_s^2)}{12} \tag{2}$$

where, ρ is a sheet density per unit depth, t_s is thickness, and L_s is length of each rigid bar. The leading body is connected to a ground by a planar joint to guarantee an in-plane motion. The planar joint has one rotational and two translational degrees of freedom. The i body is connected to the $(i+1)$ body by a revolute joint and rotational spring damper. The revolute joint has one rotational degree of freedom between two rigid bars. The relative angle of $\theta_{i(i+1)}$ is directly integrated. The torque of the rotational spring-damper is computed as following

$$\tau = -k\,\theta_{i(i+1)} - c\,\dot{\theta}_{i(i+1)} \tag{3}$$

$$k = \frac{E\, t_s^3}{12\, L_s} \tag{4}$$

$$c = \xi\, k \tag{5}$$

where, $\theta_{i(i+1)}$ and $\dot{\theta}_{i(i+1)}$ are relative angles and angular velocities of the revolute joints, and E and ξ are the young's modulus and the structural damping ratio of a sheet. The contact geometry of a sheet is described as a box and two circles as shown Figure 2. The x-axis of the body reference frame of each rigid bar is defined along longitudinal length direction and the y-axis is defined by right hand rule. The mass center of each rigid bar is located at the center point of box. In order to generate a continuous contact force, two circles are located on both sides of the box. Even thought the proposed assumed method for flexible sheet has an excellent visual appearance of the sheet under severe bending conditions, this approach shows the lack of continuity between rigid bodies, which can cause noise problems when the sheet is contact with rollers. It has also rigid leading and trailing effect of the sheet. Problems can be overcome with introducing a circular edge at leading and trailing points of each rigid bar.

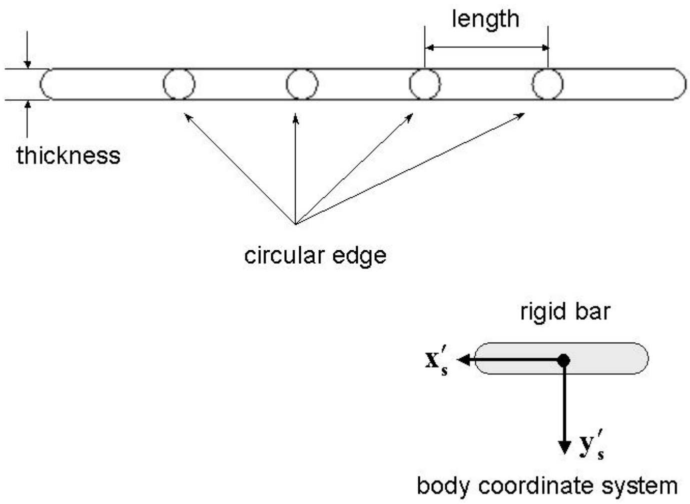

Figure 2 Contact geometry of two dimensional sheet

There can be another approach to assume flexible sheet in dynamic analysis, which employs a series of beam forces, and for the contact definitions, a rigid bar can be attached simply. One of the advantages of this approach is a natural definition of the flexible properties using the beam elements. However this approach can cause problems with the contact definitions since it has possible gaps and the lack of continuity between rigid contact bodies. The contact forces on the edges of the rigid bodies are amplified as torques applied where the rigid body is connected to the junction of two beams, and the rigid leading and trailing edges of the sheet cause unnatural behaviors.

3. CONTACT FORCE ANALYSIS

In the field of multi-body dynamics, one of the most popular approximation of the dynamic behavior of a contact pair has been that one body penetrates into the other body with a velocity on a contact point, thereafter the compliant normal and friction forces are generated between a contact pair. Figure 3 shows the schematic diagram of contact force analysis used in this investigation.

In this compliant contact force model, a contact normal force can be defined as an equation of the penetration [9], which yields

$$f_n = -k\,\delta^m - c\delta^n\,\dot{\delta} \tag{6}$$

where δ and $\dot{\delta}$ are an amount of penetration and its velocity, respectively. The spring and damping coefficients of k and c can be determined from analytical and experimental methods. The order m of the indentation can compensate the spring force of restitution for non-linear characteristics, and the order n can prevent a damping force from being excessively generated when the relative indentation is very small. As it happens, the contact force may be negative due to a large negative damping force, which is not realistic. This unnatural situation can be resolved by using the indentation exponent greater than one. The phenomenon is very important for the case of sheet contact interaction since it is very thin and light. A friction force can be determined as follows.

$$f_f = \mu(v)\,f_n \tag{7}$$

where, f_n and $\mu(v)$ are a contact normal force and a friction coefficient, respectively.

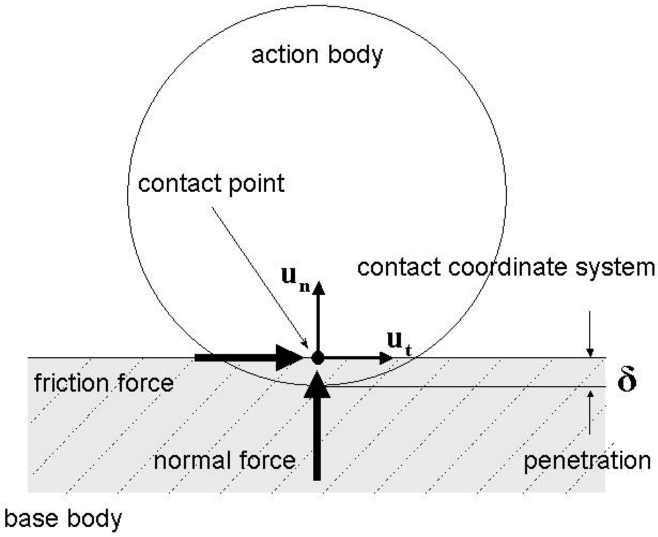

Figure 3 Contact forces between a contact pair

3.1 Kinematics Notations

The $\mathbf{X}-\mathbf{Y}$ coordinate system is the inertial reference frame and the single primed coordinate systems are the body reference frames, and the double primed coordinate system is the contact reference frame in order to define contact conditions as shown in Figure 4. The orientation and position of the body reference frame are denoted by $\mathbf{A}$ and $\mathbf{r}$, respectively.

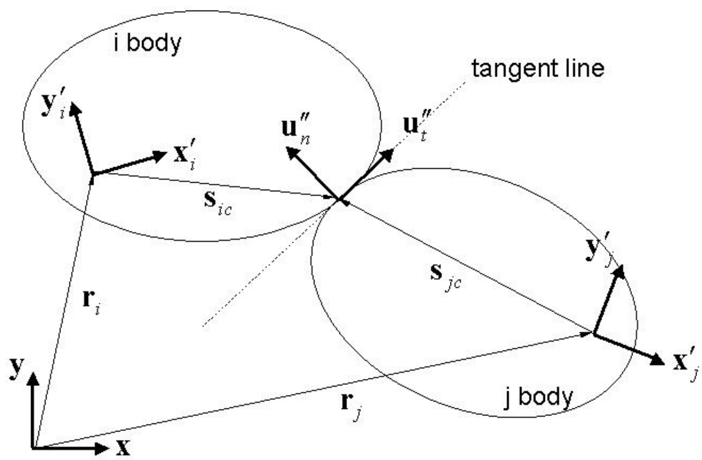

Figure 4 Kinematic notations of a contact pair

3.2 Sheet and Roller Interactions

In this investigation, two kinds of rollers are defined for the system. One is a fixed roller with one rotational degree of freedom. The fixed roller is linked to the ground with a revolute joint. The other is a movable roller, which has two degrees of freedom for a translational and a rotational motion. The movable roller is linked to rotational axis retainer (RAR) with a revolute joint and the retainer is linked to the ground with a translational joint. The contact geometry of rollers is described as a circle as shown in Figure 5

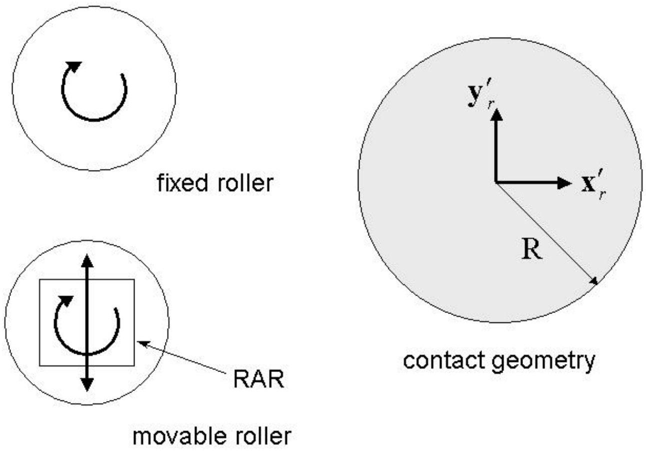

Figure 5 Definition of rollers

Two different interactions between roller and sheet are introduced in this investigation. Since the proposed flexible sheet is constructed by linear part and circular part, these are interactions between linear part and rollers, and circular part and rollers, as clearly illustrated in Figure 6

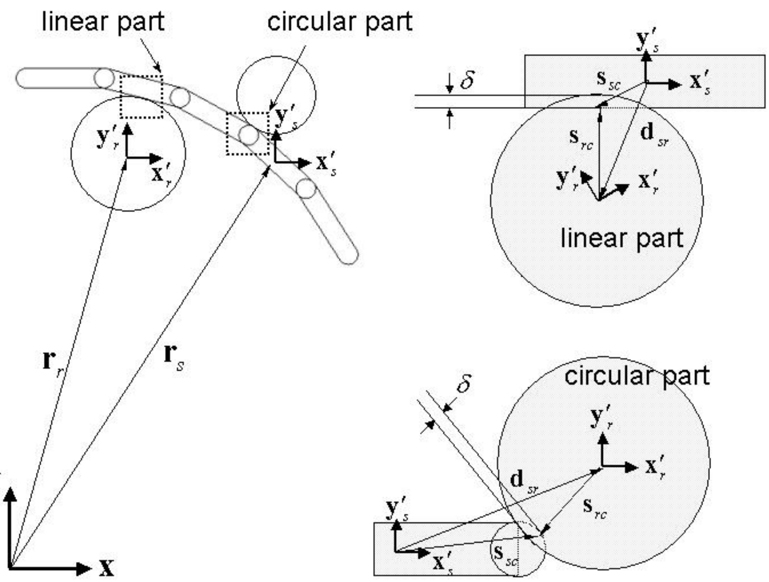

Figure 6 Sheet and roller interaction

In the case of linear part contact with rollers, the contacted penetration is determined as follows:

$$\delta = \left|\mathbf{d}_{\mathbf{sr,y}}\right| - R_r \,, \quad \mathbf{d}'_{\mathbf{sr}} = \mathbf{A_s}^{\mathbf{T}}(\mathbf{r_r} - \mathbf{r_s}) \tag{8}$$

where, $\mathbf{A_s}$ is the orientation matrix of a rigid bar, and R_r is the radius of a contacted roller, respectively. The location of contact between rigid bar and roller can be defined as follows:

$$\mathbf{s}'_{sc} = \begin{Bmatrix} \mathbf{d}'_{\mathbf{sr,x}} \\ sign(\mathbf{d}'_{\mathbf{sr},y})\, t_s / 2 \\ 0 \end{Bmatrix}, \qquad ..(9)$$

and

$$\mathbf{s}'_{\mathbf{rc}} = \mathbf{A_r}^{\mathbf{T}} \mathbf{A_s} (\mathbf{s}'_{\mathbf{sc}} - \mathbf{d}'_{\mathbf{sr}}) \qquad (10)$$

where, $\mathbf{A}_r$ and t_s are the orientation matrix of a roller and the thickness of the sheet. The relative velocity at the contact point can be determined as

$$\dot{\delta} = \mathbf{u}_\mathbf{n}^\mathbf{T} \left(\frac{d}{dt} (\mathbf{r_r} + \mathbf{A_r s'_{rc}} - \mathbf{r_s} - \mathbf{A_r s'}) \right) \qquad (11)$$

$$= \mathbf{u}_\mathbf{n}^\mathbf{T} (\dot{\mathbf{r}}_\mathbf{r} + \mathbf{A_r} \tilde{\mathbf{w}}'_\mathbf{r} \mathbf{s}'_\mathbf{rc} - \dot{\mathbf{r}}_\mathbf{s} - \mathbf{A_r} \tilde{\mathbf{w}}'_s \mathbf{s}'_\mathbf{sc})$$

$$= \mathbf{u}_\mathbf{n}^\mathbf{T} \dot{\mathbf{d}}_\mathbf{c} \qquad (12)$$

and tangential relative velocity is

$$v_t = \mathbf{u}_t^\mathbf{T} \dot{\mathbf{d}}_\mathbf{c} \qquad (13)$$

where, $\mathbf{w}'_\mathbf{r}$ and $\mathbf{w}'_\mathbf{s}$ are the angular velocities of a roller and a rigid bar with respect to each body reference frame, and $\mathbf{u}_n$ and $\mathbf{u}_t$ are the normal and tangent vectors of relative position between rigid bar and roller, respectively.

3.3 Rollers Interactions

A circle to circle contact is used to describe the interactions between circular rollers. In this circle to circle contact, the positive normal direction is same in the direction of the relative position vector between two roller center points. The tangent direction vector is determined by the right hand rule. The relative velocity and the contact forces at the contact point can be computed similarly as the sheet and roller interactions.

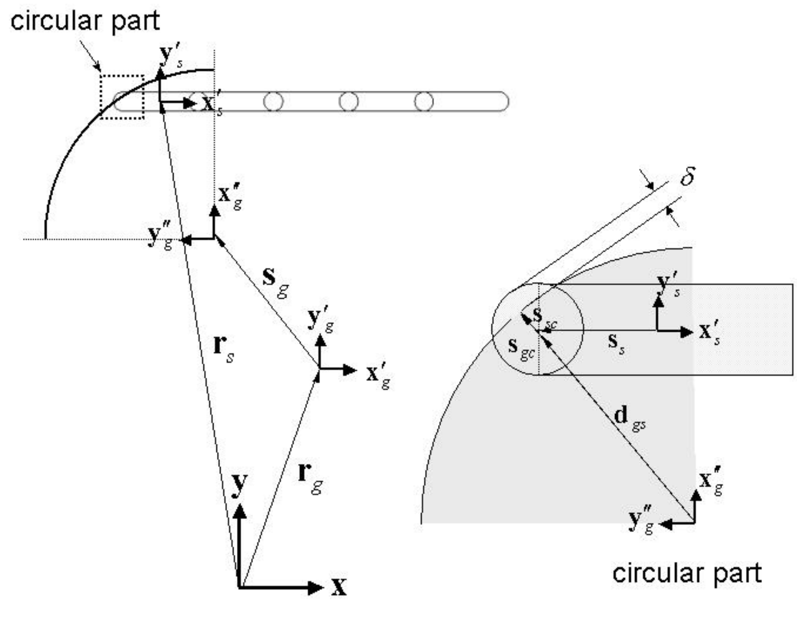

Figure 7 Sheet and arc guide interactions

3.4 Sheet and Guide Interactions

Guide has three types. Commonly used sheet guides for media transport machines can be divided into three different types, which are an arc guide with radius and angle, a linear guide with two points, and a circle guide similar to a roller. In order to avoid the complex contact detect algorithms. It is assumed that the arc and line guide are interacted with the circular part of rigid bars of the sheet. However, in the case of circle guide, both linear and circular parts of the sheet are interacted with.

As shown in Figure 7, the relative displacement between a circular edge of rigid bar and arc guide can be determined as

$$\mathbf{d}_{gs} = \mathbf{r}_s + \mathbf{A}_s \mathbf{s}'_s - \mathbf{r}_g - \mathbf{A}_g \mathbf{s}'_g \tag{14}$$

where, $\mathbf{r}_g$ and $\mathbf{A}_\mathbf{g}$ are the center position and the orientation of the guide, and the vectors of $\mathbf{s}'_\mathbf{g}$ and $\mathbf{s}'_\mathbf{s}$ are positions of the arc reference frame and the circular edge center position with respect to each body reference frame, respectively. If the vector $\mathbf{d}_{gs}$ is projected into the arc reference frame, the resultant vector can be represented as follows

$$\mathbf{d}''_{gs} = (\mathbf{A}_\mathbf{g} \mathbf{C}_\mathbf{g})^\mathrm{T} \, \mathbf{d}_{gs} \tag{15}$$

where, $\mathbf{C}_\mathbf{g}$ is the orientation matrix of the arc reference frame. The relative angle between x-axis of the arc reference frame and the resultant vector of Eq. 15 is within an arc angle, which can be written as

$$0 \le \cos^{-1}\left(\frac{\mathbf{d}''^\mathrm{T}_\mathbf{gs} \mathbf{f}''_\mathbf{g}}{\left\|\mathbf{d}''_\mathbf{gs}\right\|}\right) \le \theta_g \tag{16}$$

where, θ_g is the arc angle and $\mathbf{f}''_g$ is a constant unit vector of $[1 \quad 0 \quad 0]^\mathrm{T}$. If the condition of Eq. 16 is satisfied, the penetration between circular part of sheet and arc can be defined as follows

$$\delta = \left\|\mathbf{d}''_{gs}\right\| + t_s - R_g \tag{17}$$

where, R_g is a radius of the arc guide. The contact positions can be computed as follows.

$$\mathbf{s}''_{gc} = -R_g \mathbf{u}''_n \tag{18}$$

$$\begin{aligned} {}^g\mathbf{s}''_{sc} &= \mathbf{s}''_{gc} - \mathbf{d}''_{gs} \\ \mathbf{s}'_{sc} &= \mathbf{A}_s^\mathrm{T} \mathbf{A}_\mathbf{g} \mathbf{C}_\mathbf{g} \, {}^g\mathbf{s}''_{sc} \end{aligned} \tag{19}$$

where, $\mathbf{u}''_\mathbf{n}$ is the normal direction vector and determined

$$\mathbf{u}''_\mathbf{n} = \frac{\mathbf{d}''_{gs}}{\left\|\mathbf{d}''_{gs}\right\|} \tag{20}$$

The tangent direction vector is determined by the right hand rule, and the relative velocity at the contact point is defined as follows.

$$\begin{aligned} \dot{\mathbf{d}}_\mathbf{c} &= \frac{d}{dt}(\mathbf{r}_s + \mathbf{A}_s(\mathbf{s}'_s + \mathbf{s}'_\mathbf{sc}) - \mathbf{r}_g - \mathbf{A}_g(\mathbf{s}'_g + \mathbf{C}_\mathbf{g}\mathbf{s}''_{gc})) \\ &= \dot{\mathbf{r}}_s + \mathbf{A}_s \tilde{\mathbf{w}}'_s (\mathbf{s}'_s + \mathbf{s}'_\mathbf{sc}) - \dot{\mathbf{r}}_g - \mathbf{A}_g \tilde{\mathbf{w}}'_g (\mathbf{s}'_g + \mathbf{C}_\mathbf{g}\mathbf{s}''_{gc}) \end{aligned} \tag{21}$$

where, $\mathbf{w}'_g$ and $\mathbf{w}'_s$ is the angular velocities of guide and a bar with respect to each body reference frame, respectively. The contact forces at the contact point can be computed similarly as described in the sheet and roller interactions.

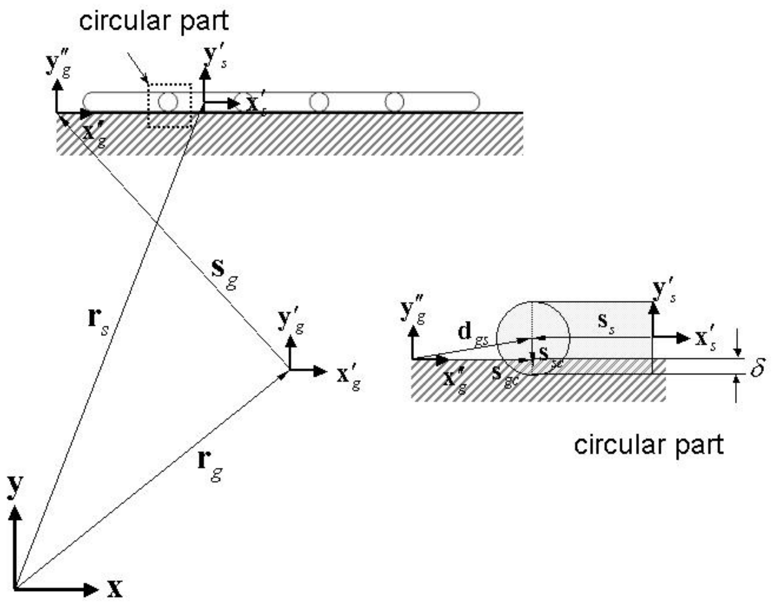

Figure 8. Sheet and line guide interactions

The sheet and line guide interactions are illustrated in Figure 8 clearly. If the $\mathbf{x}$ component of the vector $\mathbf{d}_{gs}$ defined in the double primed line guide reference frame is the range of guide length, simple circle and line contact algorithm is used in this investigation. After definitions of penetration and its derivative, the contact force is created to restitute each body as similar as previous interactions between sheet and guides.

4. EQUATIONS OF MOTION

Since the multibody sheet system interacts with the roller and guide components through the contact forces and adjacent rigid bars are connected by revolute joint and rotational spring damper forces as shown in Figure 9, each sub-rigid bar in the sheet system has one degree of freedom which is represented by one rotational coordinates and the leading body has three free coordinates. The equations of motion of the sheet system that employs the velocity transformation defined by Bae [8] are given as follows:

$$\mathbf{B}^{\mathbf{T}}\mathbf{M}\mathbf{B}\ddot{\mathbf{q}}_i = \mathbf{B}^{\mathbf{T}}(\mathbf{Q} - \mathbf{M}\dot{\mathbf{B}}\dot{\mathbf{q}}_i^r) \tag{22}$$

where $\mathbf{q}_i^r$, $\mathbf{B}$ and $\dot{\mathbf{q}}$ are relative independent coordinates, velocity transformation matrix, and Cartesian velocities of the media feeding system, and $\mathbf{M}$ is the mass matrix, and $\mathbf{Q}$ is the generalized external and internal force vector of the media feeding system, respectively. The velocity transformation matrix $\mathbf{B}$ of the sheet is more explicitly as

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{012} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{B}_{121}\mathbf{B}_{012} & \mathbf{B}_{122} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{B}_{231}\mathbf{B}_{121}\mathbf{B}_{012} & \mathbf{B}_{231}\mathbf{B}_{122} & \mathbf{B}_{232} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{B}_{(n-1)n1}\cdots\mathbf{B}_{012} & \mathbf{B}_{(n-1)n1}\cdots\mathbf{B}_{122} & \mathbf{B}_{(n-1)n1}\cdots\mathbf{B}_{232} & & \mathbf{B}_{(n-1)n2} \end{bmatrix}$$

where the recursive velocity and virtual relationship for a pair of rigid bars are obtained [8] as

$$\mathbf{Y}_i = \mathbf{B}_{(i-1)i1}\mathbf{Y}_{(i-1)} + \mathbf{B}_{(i-1)i2}\dot{\mathbf{q}}_{(i-1)i} \tag{23}$$

and $\mathbf{q}_{(i-1)i}$ denotes the relative coordinate vector. It is important to note that matrices $\mathbf{B}_{(i-1)i1}$ and $\mathbf{B}_{(i-1)i2}$ are only functions of the $\mathbf{q}_{(i-1)i}$.

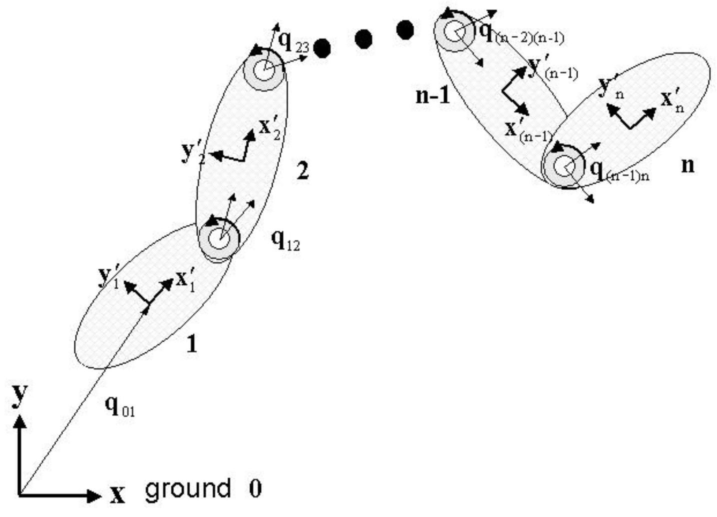

Figure 9 Kinematic relationships between rigid bars of the sheet

5. NUMERICAL RESULTS

The proposed algorithm is implemented and a film-feeding problem is solved to demonstrate the efficiency and validity of the proposed method.

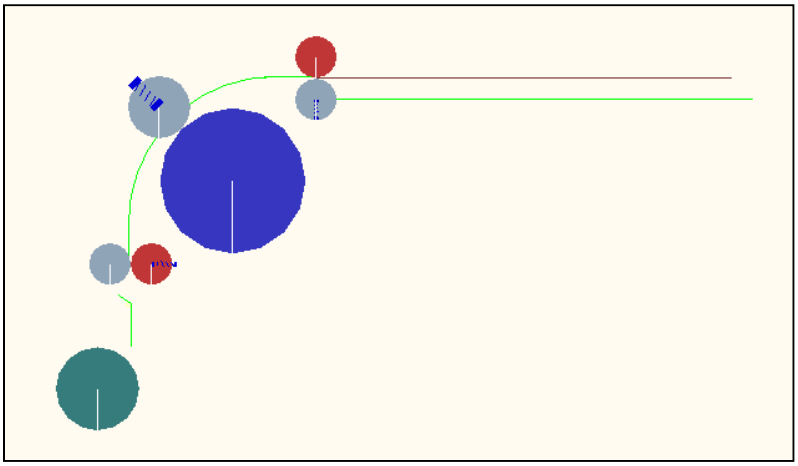

Figure 10 Film feeding machine

The system has 29 degrees of freedom and consists of four fixed rollers and three movable rollers, five line guides, one arc and circle guide and one sheet of film shown in Figure 10. The sheet is modeled by using 20 rigid bars. The density and Young's modulus of sheet are 2.2e-6(kg/mm^3) and 2250(N/mm^2), respectively. And the thickness and length of sheet are 0.5(mm) and 200(mm), respectively.

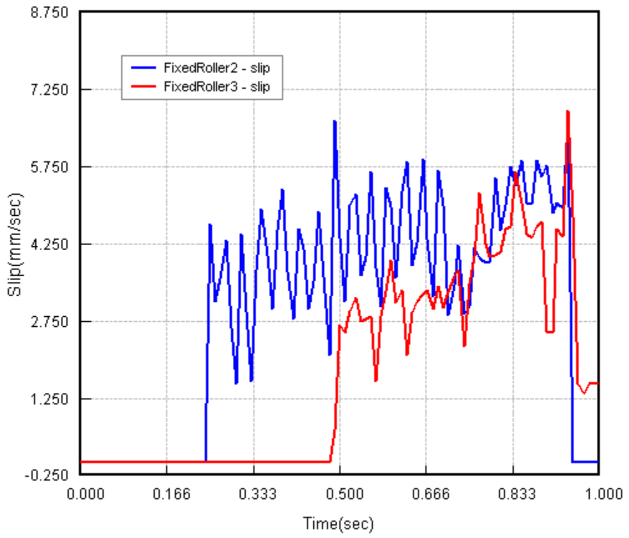

Figure 11 Slip between rollers and sheet

The film goes through a path while contacting the roller pairs. The circumferential speed of each driving roller is 300($mm/\sec$). The slip velocities between driving rollers and the sheet are shown in Figure 11. The path of first, second and third segment bodies of the thin film are plotted as shown in Figure 12. The x and y axes of the plot are displacements measured in the directions of x and y axes in the global reference frame, respectively.

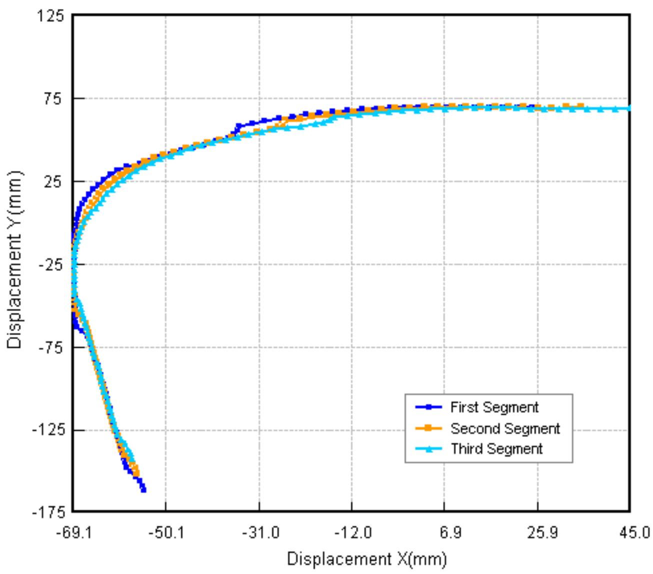

Figure 12 Path of segmented bodies of film

The analysis was performed on an IBM compatible computer (PIII-933Mhz) and took about 60 sec. per 1 sec. for simulation.

6. SUMMARY AND CONCLUSION

The dynamics and modeling techniques of two dimensional media transport system is investigated in this paper. The flexible sheet is divided by finite number of rigid bars. Linear motions are constrained in order to allow rotations between the rigid bars of the sheet. Rotational spring damper force is applied for the reflection flexible stiffness of the sheet. From previous empirical measurements in manufacturing process effective stiffness and damping coefficients are substituted in this investigation. Compliant contact force model is used for the interactions between sheet rollers, and guides. Kinematics notations of the contact search algorithms for the media transport system are clearly represented. A simple film feeding example is represented in this investigation and manufacture [3] confirms that simulation results have very good agreement with experimental measurements. The media transport system manufactures have rely on trial error techniques for the design of their core mechanisms, however the proposed method by employing multibody dynamics in this paper can reduce many difficulties at the early design stage.

Acknowledgement

This research was supported by Korea Science and Engineering Foundation (KOSEF R0 1-2000-000246-0).

References

1. H. J. Cho, and J. H. Choi, 2001, “2DMTT development specification” Technical report, FunctionBay Inc.
2. Ted Diehl, 1995, “Two dimensional and three dimensional analysis of nonlinear nip mechanics with hyperelastic material formulation” Ph. D. Thesis, University of Rochester, Rochester, NewYork
3. Tsuyoshi Ashida, 2002, “Technical meeting” Miyanodai Technology Development Center, Fuji Film Inc. Japan
4. http://www.yanabelab.nagaokaut.ac.jp/
5. http://www.me.psu.edu/research/bension.html
6. http://www.engext.okstate.edu/info/WWW-WHRC.htm
7. Shin, K. H., 1991, “Distributed Control of Tension in Multi-Span Web Transport Systems “, Ph. D. Thesis Oklahoma State Univ.
8. D. S. Bae, J. M. Han, and H. H. Yoo, 1999, “A Generalized Recursive Formulation for Constrained Mechanical System Dynamics”, Mech. Struct. And Machines, Vol. 27, No 3, pp 293-315
9. Lankarani H. M. and Nikravesh P. E., 1994, “Continuous Contact Force Models for Impact Analysis in Multibody Systems”, Journal of Nonlinear Dynamics, Kluwer Academic Publishers, Vol. 5, pp 193-207